Graph Theory Glossary

Chris Caldwell © 1995

This glossary is written to supplement the Interactive Tutorials in Graph Theory. Here we define the terms that we introduce in our tutorials--you may need to go to the library to find the definitions of more advanced terms. Please let me know of any corrections or suggestion!

[A B C D E F G H I J K L M N O P Q R S T U V W X Y Z]

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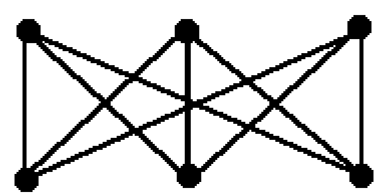
adjacent
Two vertices are adjacent if they are connected by an edge.

arc
A synonym for edge. See graph.

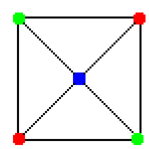
articulation point
See cut vertices.

bipartite



A graph is **bipartite** if its vertices can be partitioned into two disjoint subsets U and V such that each edge connects a vertex from U to one from V. A bipartite graph is a **complete bipartite** graph if every vertex in U is connected to every vertex in V. If U has *n* elements and V has *m*, then we denote the resulting complete bipartite graph by $K_{n,m}$. The illustration shows $K_{3,3}$. See also complete graph and cut vertices.

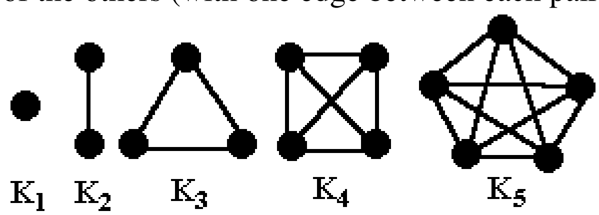
chromatic number



The chromatic number of a graph is the least number of colors it takes to color its vertices so that adjacent vertices have different colors. For example, this graph has chromatic number three.
When applied to a map this is the least number of colors so necessary that countries that share nontrivial borders (borders consisting of more than single points) have different colors. See the Four Color Theorem.

circuit
A circuit is a path which ends at the vertex it begins (so a loop is an circuit of length one).

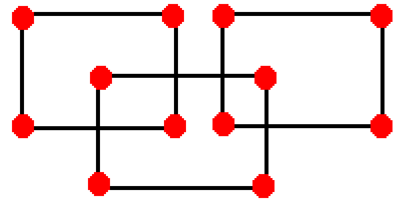
complete graph
A complete graph with *n* vertices (denoted K_n) is a graph with *n* vertices in which each vertex is connected to each of the others (with one edge between each pair of vertices). Here are the first five complete graphs:



component
See connected.

connected

A graph is connected if there is a path connecting every pair of vertices. A graph that is not connected can be divided into **connected components**(disjoint connected subgraphs). For example, this graph is made of three connected components.

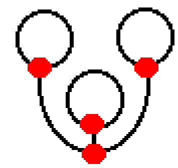


cut vertex

A cut vertex is a vertex that if removed (along with all edges incident with it) produces a graph with more connected components than the original graph. See connected.

degree

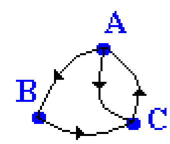
The degree (or valence) of a vertex is the number of edge *ends* at that vertex. For example, in this graph all of the vertices have degree three.



In a digraph (directed graph) the degree is usually divided into the in-degree and the out-degree (whose sum is the degree of the vertex in the underlying undirected graph).

digraph

A digraph (or a **directed graph**) is a graph in which the edges are directed. (Formally: a digraph is a (usually finite) set of vertices V and set of *ordered* pairs (*a*,*b*) (where *a, b* are in V) called edges. The vertex *a* is the **initial vertex** of the edge and *b* the **terminal vertex**.



directed graph

See digraph.

edge

See graph.

Four Color Theorem

Every planar graph can be colored using no more than four colors.

graph

Informally, a graph is a finite set of dots called **vertices** (or **nodes**) connected by links called **edges** (or **arcs**). More formally: a **simple graph** is a (usually finite) set of vertices V and set of unordered pairs of distinct elements of V called edges.

Not all graphs are simple. Sometimes a pair of vertices are connected by multiple edge yielding a multigraph. At times vertices are even connected to themselves by a edge called a loop, yielding a pseudograph. Finally, edges can also be given a direction yielding a directed graph (or digraph).

in-degree

The in-degree of a vertex *v* is the number of edges with *v* as their terminal vertex. See also digraph and degree.

initial vertex

See digraph.

isolated

A vertex of degree zero (with no edges connected) is isolated.

Kuratowski's Theorem

A graph is nonplanar if and only if it contains a subgraph homeomorphic to $K_{3,3}$ or K_5.

length

For the length of a path see path.

loop

A loop is an edge that connects a vertex to itself. (See the illustration for degree which has a graph with three loops.) See pseudograph for a formal definition of loop.

multigraph

Informally, a multigraph is a graph with multiple edges between the same vertices. Formally: a **multigraph** is a set V of **vertices** along, a set E of **edges**, and a function f from E to $\{\{u,v\}|u,v$ in V; u,v distinct$\}$. (The function f shows which vertices are connected by which edge.) The edges r and s are called **parallel** or **multiple** edges if $f(r)=f(s)$. See also graph and pseudograph.

multiple edge

See multigraph.

node

A synonym for vertex. See graph.

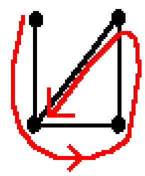
out-degree

The out-degree of a vertex v is the number of edges with v as their initial vertex. See also digraph and degree.

parallel edge

See multigraph.

path



A path is a sequence of consecutive edges in a graph and the length of the path is the number of edges traversed. (This illustration shows a path of length four.)

pendant

A vertex of degree one (with only one edge connected) is a pendant edge.

planar

A graph is planar if it can be drawn on a plane so that the edges intersect only at the vertices. (For example, of the five first complete graphs all but the fifth, K_5, is planar.)

pseudograph

Informally, a pseudograph is a graph with multiple edges (or loops) between the same vertices (or the same vertex). Formally: a **pseudograph** is a set V of **vertices** along, a set E of **edges**, and a function $f$from E to $\{\{u,v\}|u,v$ in V$\}$. (The function f shows which vertices are connected by which edge.) An edge is a **loop** if $f(e) = \{u\}$ for some vertex u in V. See also graph and multigraph.

terminal vertex

See digraph.

undirected edge

Edges in graphs are undirected (as opposed to those in digraphs).

valence

See degree.

vertex

See graph.

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