Eccentricity, Center, Radius, Diameter

Let *G* be a graph and *v* be a vertex of *G*. The *eccentricity* of the vertex *v* is the maximum distance from *v* to any vertex. That is, *e(v)*=max*{d(v,w):w* in *V(G)}*.

The *radius* of *G* is the minimum eccentricity among the vertices of *G*. Therefore, *radius(G)*=min*{e(v):v* in *V(G)}*.

The *diameter* of *G* is the maximum eccentricity among the vertices of *G*. Thus, *diameter(G)*=max*{e(v):v* in *V(G)}*.

The *girth* of *G* is the length of a shortest cycle in *G*.

The *center* of *G* is the set of vertices of eccentricity equal to the radius. Hence, *center(G)={v* in *V(G):e(v)=radius(G)}*.

A tree *T* has |*center(T)*|*=1* or |*center(T)*|*=2*. If |*center(T)*|*=1*, the tree is called *central*. If |*center(T)*|*=2*, the tree is called *bicentral*. A vertex transitive graph *G* has *center(G)=V(G)*. For any graph *G*, the diameter is at least the radius and at most twice the radius. For a tree *T*, *diameter(T)=2radius(T)-1*, if *T* is bicentral, and *diameter(T)=2radius(T)*, if *T* is central.

A *(m,n)-cage* is an *m*-regular graph with girth *n* and, subject to this, with the least possible number of vertices.

Hoffman-Singleton Theorem. Let *G* be a *k*-regular graph, with girth *5* and diameter *2*. Then, *k* is in *{2,3,7,57}*.

For *k=2*, the graph is C_5. For *k=3*, the graph is the Petersen graph. For *k=7*, the graph is called the Hoffman-Singleton graph. Finding a graph for *k=57* is still open, as far as I know.

Hoffman and Singleton proved more: There is an obvious lower bound on *f(m,n)*, the number of vertices in an *(m,n)*-cage. That is,

$$(m-2)f(m,n) >= m(m-1)^r-2 \text{ if } n=2r+1$$

and

$$(m-2)f(m,n) >= 2(m-1)^r-2 \text{ if } n=2r.$$

If n >= 6 and m >= 3 and if equality holds, then n is in {6,8,12}.

Last modified Februrary 1, 1999, by S.C. Locke. How to contact me.